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Complex mapping as a boundary treatment for stability of compressible flows

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Abstract

We focus our attention on the stability computations of compressible flows in subsonic regime. The use of artificial boundary conditions in compressible simulations is mandatory since truncated computational domains induce nonphysical wave reflections. Moreover, it has been observed that, even for incompressible flows, artificial boundary conditions are required to reduce the computational domain used for the solution of the linearized Navier-Stokes equations [1]. We found that most of the investigations reported in literature adopted the sponge regions as boundary conditions [2]. However, a valid alternative is offered by the Perfectly matched layer (PML). Unfortunately both of these methods increase the computational burden of the simulation because they require, respectively, large domains and the introduction of additional equations and unknowns [3].
Here, we propose to adopt a complex mapping to dump the wave reflections. This choice preserves the dimension of the original system. This property is very important in parametric studies of eigenproblems in two and three spatial dimensions. We show the application of simple complex mappings in two different flow configurations. The first example is the forced and free acoustic flow through a duct where a comparison against PML and analytical boundary conditions is carried out. The computed impedance with this novel methodology matches those with PML and analytical conditions in a given range. Complex mapping prove to be more robust than Sommerfeld boundary condition for the free problem. Secondly, we consider the birdcall configuration. In this configuration the use of complex mapping is strongly suggested, with respect to sponge techniques, because the low Mach numbers involved in the computations imply a large sponge region. Furthermore, the use of complex mapping allows to identify discrete modes at the threshold of a bifurcation thanks to the tilt of the continuous branch of the spectrum [4].

Complex mapping technique Complex mapping is based on *analytical continuation* of a given PDE or system of PDEs. In our study, the Navier Stokes equations are analytically continuated into complex spatial coordinates where fields are exponentially decaying. CM is based on an *analytical continuation* of the solution of a PDE to complex coordinates, where oscillating or diverging waves become exponentially decaying waves outside the region of interest. In such a way, we could propose a general complex mapping technique that transforms a numerical Euclidean space $\mathbb{R}^N$ whose coordinates are X_i, to physical coordinates x_i, for $i = 1, 2, ..., N$

$$\mathcal{G}_x : \mathbb{R} \to \mathbb{C} \qquad \text{such that} \qquad X \mapsto \mathcal{G}_x(X) = h(X)\Big[1 + i\gamma_c g(X)\Big] \tag{1}$$

where $g(X) = \int_{X_0}^{X} \tau(x')dx'$ is a smooth function that controls the transition from the real line to a specific direction $z = (1, i\gamma_c)$ of the complex plane. The parameter γ_c dictates the complex direction of the coordinate x_i, that is it controls the absorption of a given wave. For a given spatial mode to be spatially evanescent it needs to satisfy $0 < \arg(k) + \arctan(\gamma_c) < \pi$

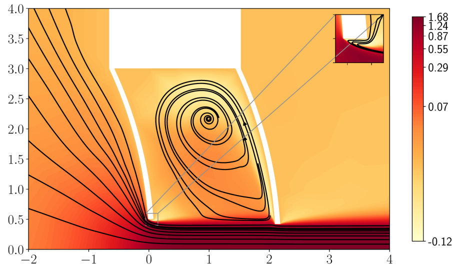

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Figure 1: A subfigure

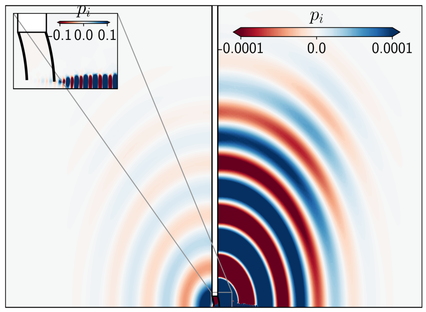

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Figure 2: A subfigure

Figure 3: A figure with two subfigures

Numerical results - Birdcall We choose to demonstrate our method on the problem of the flow passing through a circular hole in a plate. First of all we report in Table 1 the computational times obtained by using the sponge and the complex mapping method. We found a dramatic reduction of the run-time using the same spatial resolution in the interesting flow region (i.e. near the hole). The reported times take into account the computation of the baseflow and the leading eigenvalue at $Re = 400$ and $M = 0.05$.

Mesh	Methodology	ω	Time (s)
$\mathcal{M}_1$	Sponge	$4.7574 + 0.0792i$	83944 s
$\mathcal{M}_2$	Complex Mapping	$4.6922 + 0.0666i$	1655 s
$\mathcal{M}_3$	Complex Mapping	$4.7151 + 0.0945i$	1421 s
$\mathcal{M}_4$	Complex Mapping	$4.7051 + 0.0747i$	669 s

Table 1: Comparison of the time required to compute the base flow and the leading eigenvalue.

An example of base flow and pressure global mode fields are depicted in fig. 4.

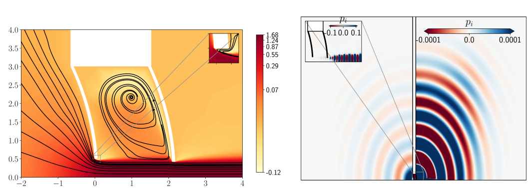

Figure 4: Some features of stability analysis of the birdcall at $Re = 1600$ and $M = 0.05$. At a) the base flow is shown, colors represent streamwise velocity U_x, whereas streamlines are shown in black. b) On the top real part of pressure mode 4 is represented p_r and on the bottom the imaginary part of the vorticity ω_i mode 4. c) and d) show the acoustic pressure fluctuations of mode 2 and 4.

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