Matrix-free linear stability analysis of fluid-structure interactions with Immersed-Boundary method

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Summary. In this paper a novel time-stepping approach for the linear stability analysis of elastic structures interacting with incompressible viscous flows is presented. The method is based on the coupling of an iterative Krylov-subspace algorithm for the solution of large eigenvalue problems within an Immersed-Boundary (IB) framework, to obtain a matrix-free global stability solver which extracts the necessary information directly from Direct Numerical Simulation (DNS).
In this work, the proposed approach is explained and validated with respect to test cases involving elastically-mounted rigid bodies immersed in open flows.

Introduction

The interactions of elastic structures with incompressible flows have been the subject of numerous works in recent years, due to their dynamically rich and complex behaviour. Despite the non-linear nature of these phenomena, the onset of the transition from a state of the system to another one can be predicted via a linear stability analysis. However, even with high-performing numerical tools, the global stability analysis of a fluid–structure system remains a prohibitive task due to considerable implementation difficulties; in particular, the linearisation of interface terms is often cumbersome.
This work presents an approach to perform the linear stability analysis of fluid-structure configurations which is incorporated into a validated FSI solver based on a Moving-Least-Squares Immersed boundary [1]. This technique has been largely used to explore the behaviour of fully coupled FSI applications involving large displacements, while linear stability analyses, on the other hand, are usually performed using mesh-deforming techniques. Although the latter methods are often chosen for two-dimensional cases, their implementation is often too expensive in three-dimensional applications. Furthermore, the extension to complex multi-body configurations is non-trivial.
In this work, we adopt the approach followed by Mack and Schmid [2] for the global stability analysis of compressible flows and extend it to the coupled physics problem.

Methodology

We investigate the dynamics of elastically-mounted rigid bodies immersed in a Newtonian viscous fluid with homogeneous density. The flow is governed by the Navier-Stokes equations, here written in an Immersed-Boundary formulation,

$$\begin{cases} \dfrac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \dfrac{1}{Re}\Delta\mathbf{u} + \mathbf{f}, \quad \text{in} \quad \Omega \\ \nabla\cdot\boldsymbol{u} = 0, \quad \text{in} \quad \Omega \end{cases} \tag{1}$$

where $\boldsymbol{u}$ and p denote the fluid velocity and pressure respectively, Ω is the physical region occupied by the fluid and $\mathbf{f}$ is the volume force arising from the IB treatment. The immersed bodies are modeled as mass-spring-damper systems,

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{F}_{nl}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{F}(t), \tag{2}$$

where $\boldsymbol{x}$ is displacement vector, $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the lumped mass, damping and stiffness matrix respectively, and $\boldsymbol{F}_{nl}$ takes into account the non-linear contributions. The forcing vector $\boldsymbol{F}(t)$ provides the resultant force on the body which, in the absence of gravity and other volume force components, reduce to the hydrodynamic load on the immersed surfaces. Collecting fluid and solid variables into the state vector $\boldsymbol{q}$, the coupled problem can be reformulated as

$$\frac{\partial \boldsymbol{q}}{\partial t} = \boldsymbol{R}(\boldsymbol{q}), \tag{3}$$

where $\boldsymbol{q}$ is the state vector, containing all the degrees of freedom of the problem, and $\boldsymbol{R}$ is the nonlinear evolution operator. The stability of the system in the vicinity of a steady solution of (3), $\boldsymbol{q}_b$, is given by a generalized eigenvalue problem for the Jacobian operator $\boldsymbol{J}$, who is an unknown quantity in the general case. Therefore, following Eriksson and Rizzi [3], we linearise the system numerically, approximating the evolution in time of a small perturbation $\boldsymbol{q}_p$ via the second-order accurate finite difference

$$\boldsymbol{q}_p(t_0 + n\Delta t) = \frac{\boldsymbol{q}_+ - \boldsymbol{q}_-}{2\epsilon}, \tag{4}$$

where $\boldsymbol{q}_+$ and $\boldsymbol{q}_-$ are outcomes of two disjoint advancement carried out by the DNS-IB solver:

$$\boldsymbol{q}_+ = \boldsymbol{F}(\boldsymbol{q}_b + \epsilon\boldsymbol{q}_p(t_0), n\Delta t), \tag{5}$$

$$\boldsymbol{q}_- = \boldsymbol{F}(\boldsymbol{q}_b - \epsilon\boldsymbol{q}_p(t_0), n\Delta t). \tag{6}$$

Recalling that the evolution of a small perturbation $\boldsymbol{q}'$ of the base state is given by

$$\boldsymbol{q}'(t_0 + T) = e^{\boldsymbol{J}T}\boldsymbol{q}'(t_0), \tag{7}$$

we recognize that the finite difference (4) approximates the action of the exponential transformation of the Jacobian matrix on the perturbation vector. Thus, we can compute a set of the least stable eigenvalues of the system by means of an iterative algorithm such as those belonging to the general class of Krylov subspace methods.
In the present work, the approximation of the leading eigenvalues of the system is computed by using the *Implicity Restarted Arnoldi Method* (IRAM) as implemented in the ARPACK open source package [4]. The steps of the algorithm are listed below:

1. The base flow is computed via the *BoostConv* [7] stabilization procedure;
2. Arnoldi iterations are performed until convergence is reached: $(k = 1, 2, ...)$
   (a) Vector $\boldsymbol{q}_p^k$ is generated
   (b) Reverse communication to the DNS-IB solver gives $\boldsymbol{q}_p^k(n\Delta t) = \frac{\boldsymbol{q}^+ - \boldsymbol{q}^-}{2\epsilon}$
   (c) Convergence of the desired Ritz pairs is checked
3. A logarithmic transformation is performed to recover the eigenvalues of the Jacobian matrix.

Results and discussion

The presented methodology has been validated with respect to several literature cases involving rigid bluff bodies interacting with flows with low Reynolds numbers. As a proof of concept, figure 1 shows the growth rate and frequency of the first two eigenvalues for the case of a elastically-mounted circular cylinder, undergoing vortex-induced vibrations in a flow with $Re = 60$. The cylinder is allowed to move only in the direction orthogonal to the unperturbed flow.
Given its matrix-free and Immersed-Boundary formulation, the method is capable of coping with complex geometries,

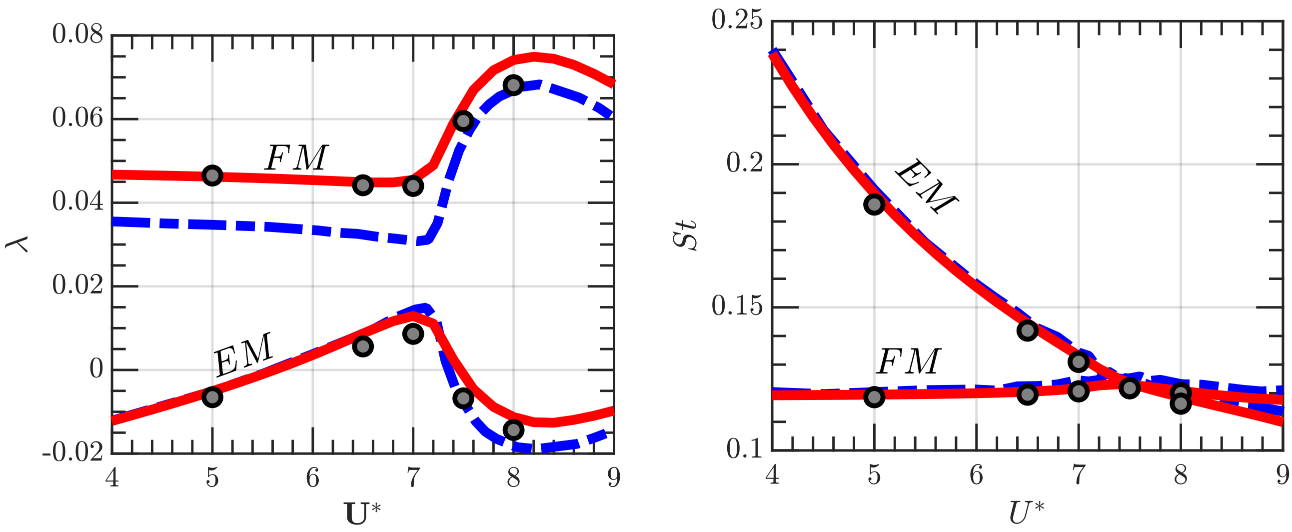

Figure 1: Results of the linear stability analysis of the VIV cylinder with $Re = 60$ and $m^* = 20$. Left figure: growth rate of the two least stable eigenmodes with U^*; Right figure: Strouhal number variation of the same eigenvalues with U^*. Red line: results from [6]; blue dotted line: results from [5]; black circles: present results.

multibody systems and large-scale inhomogeneous three-dimensional flows.

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