Bifurcation scenario in the wake of a rotating cylinder in uniform flow

A dynamical system perspective

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Introduction

Flow configuration

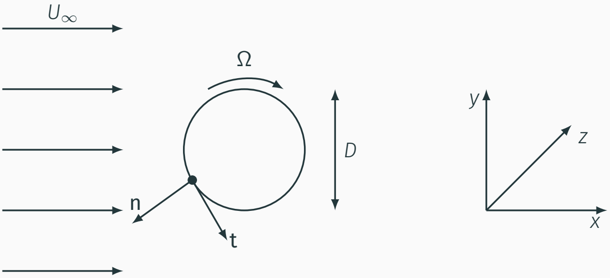


Figure 1: Diagram of the rotating cylinder in uniform flow

Flow motion governed by the incompressible Navier–Stokes equations

$$\frac{\partial \mathsf{U}}{\partial t} + \mathsf{U} \cdot \nabla \mathsf{U} = -\nabla P + \nabla \cdot \tau(\mathsf{U}) \tag{1a}$$

$$\nabla \cdot \mathsf{U} = 0 \tag{1b}$$

with two parameters $Re = \frac{U_\infty D}{\nu}$ and the rotation rate $\alpha = \frac{\Omega D}{2U_\infty}$.

Review of previous studies

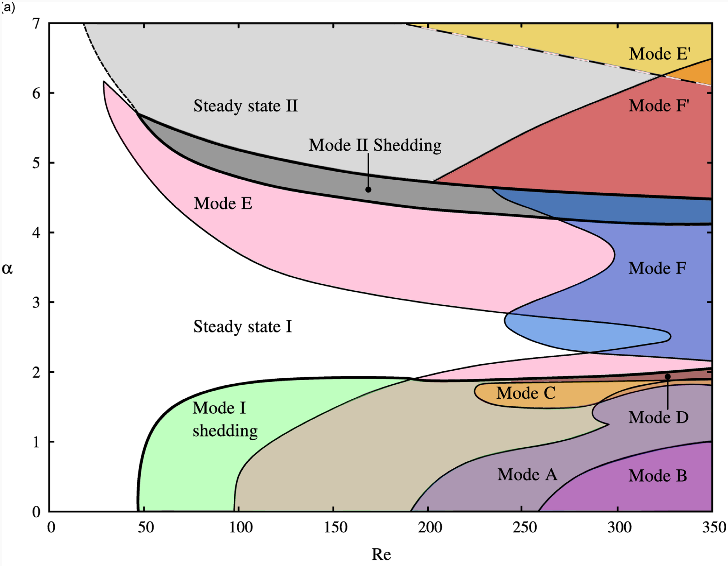


Figure 2: Neutral curves of stability. Extracted from [4].

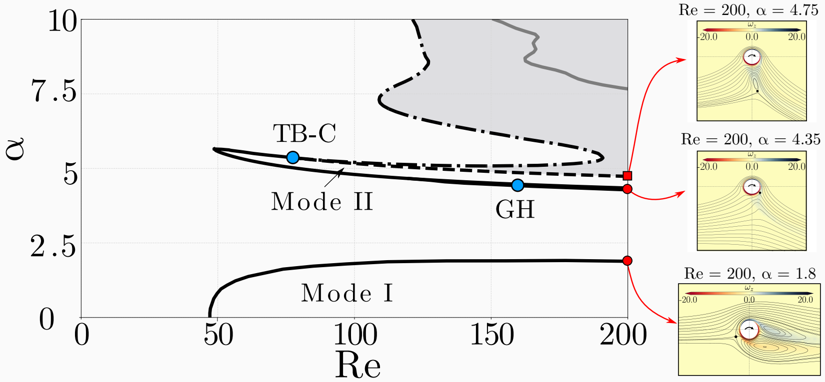


Figure 3: Neutral curves of stability. Extracted from [5].

Existence of multiple solutions

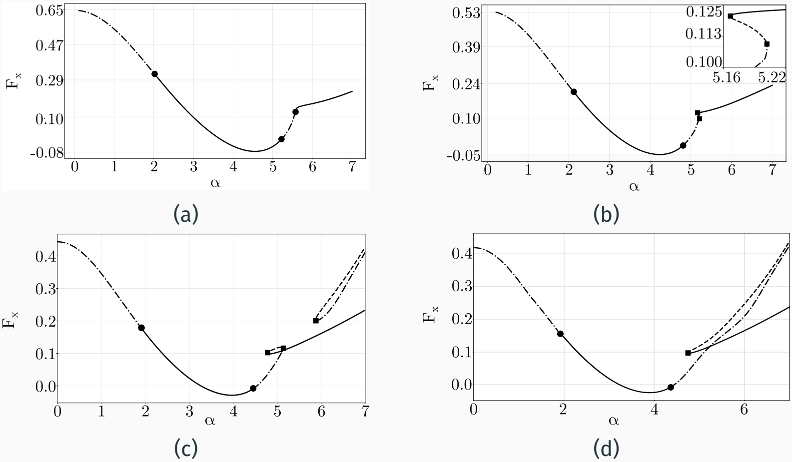


Figure 4: Evolution of the horizontal force F_x as a function of the rotation rate α for four Reynolds numbers, (a) $Re = 60$, (b) $Re = 100$, (c) $Re = 170$ an (d) $Re = 200$.

Results

How do multiple solutions arise? A first organizing center

The transition occurring for $Re \approx 75$ and $\alpha \approx 5.4$ is characterised by the end of the Hopf curve (H_-) at a fold curve (F_+)

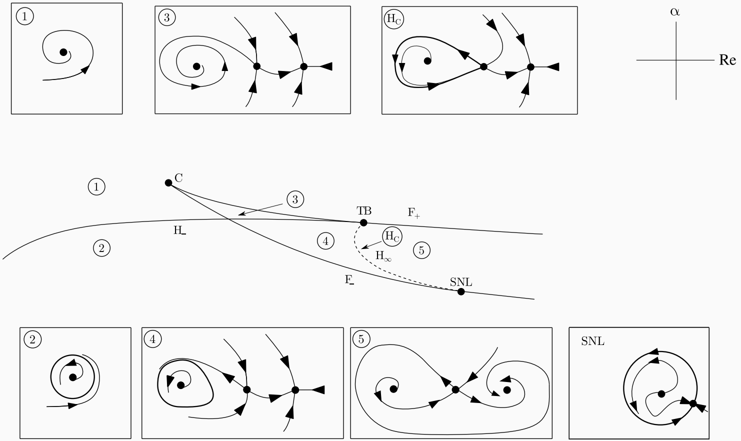


Figure 5: Neutral curves of stability. Extracted from [5].

How do multiple solutions arise? A first organizing center (II)

The dynamical behaviour of the system can thus be expected to be well predicted using the normal form describing the universal unfolding of the *codimension-3 planar bifurcation*, also called *generalized TB bifurcation*, cf [2] and [3, Ch. 8.3].

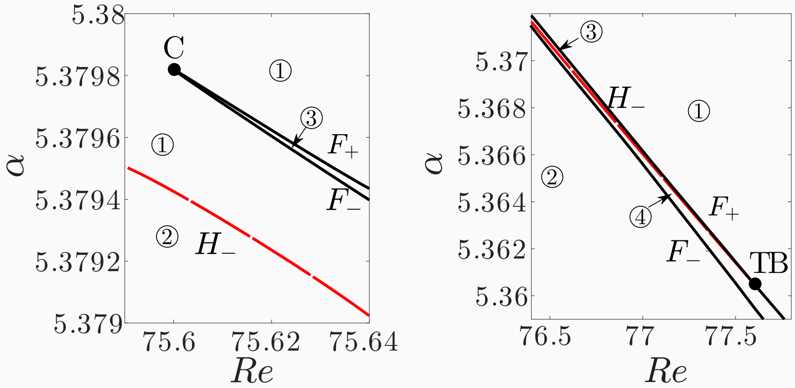
The normal form can be written as follows:

$$\frac{dy_1}{dt} = y_2 \tag{2a}$$

$$\frac{dy_2}{dt} = \beta_1 + \beta_2 y_1 + \beta_3 y_2 + \epsilon y_1^3 + c_1 y_1 y_2 - y_1^2 y_2 \tag{2b}$$

where (β_1, β_2 and β_3) are unfolding parameters (mapped from the physical parameters (Re, α)), c_1, ϵ (which can be rescaled to ± 1) are fixed coefficients which depend on the nonlinear terms of the underlying system. Note that this normal form generalizes both the normal form of the standard TB bifurcation (which is recovered for $\beta_1(Re, \alpha) = 0$) and the one of the fold bifurcation (which is recovered for $\beta_3(Re, \alpha) = 0$).

Numerical results in the vicinity of TB and cusp bifurcations

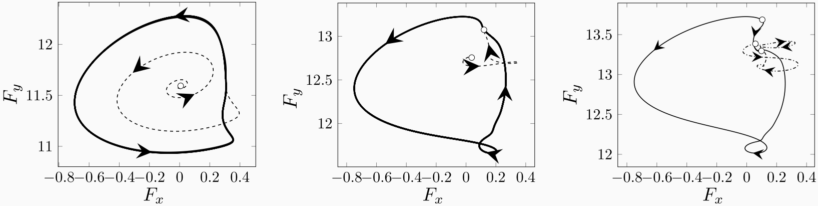


(a) Zoom in the region of cusp bifurcation.

(b) Zoom in the region of Takens Bogdanov bifurcation.

Figure 6: Zooms of figure 5 in the vicinity of the *C* and *TB* codimension-2 points. Black solid lines denote fold bifurcations $F_{\pm}$, long dashed (red online) line is used for the Hopf bifurcation line H_- and the short dashed curve denotes the local change from stable focus to stable node.

A bifurcation near the Saddle-Node Loop bifurcation (SNL)



(a) (b) (c)

Figure 7: Phase portrait of the dynamics of the rotating cylinder at $Re = 170$ for three values of the rotation rate α.

A second organizing center – Generalized Hopf bifurcation

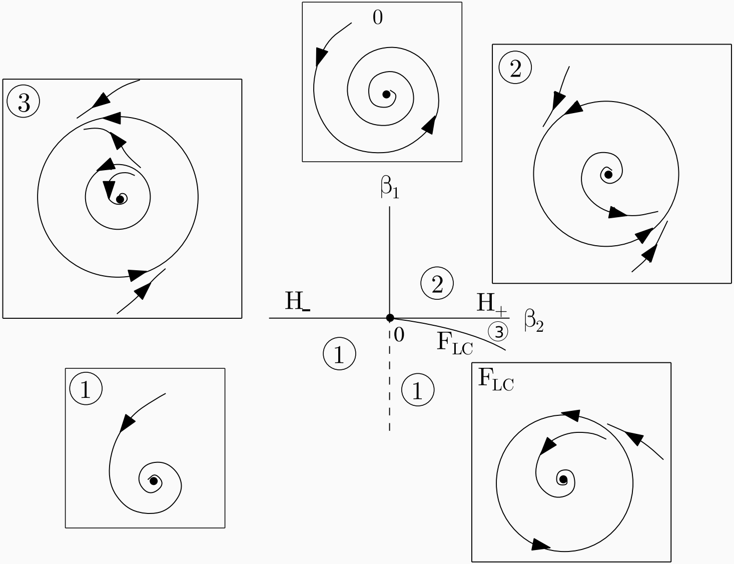
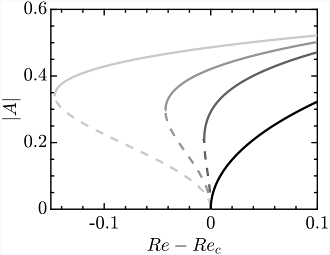


Figure 8: Generalized Hopf bifurcation. Phase portraits. Extracted from [5].

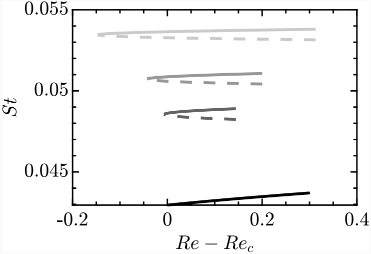
A second organizing center – Generalized Hopf bifurcation (II)

The complex amplitude A evolves as

$$\frac{dA}{dt} = (i\omega_0 + \epsilon^2\lambda_0 + \epsilon^4\lambda_1)A + (\nu_{1,0} + \epsilon^2\nu_{1,1})|A|^2 A + \nu_{2,0}|A|^4 A \qquad (3)$$



(a)



(b)

Figure 9: Amplitude and Strouhal number of stable (solid line) and unstable (dashed line) limit cycles for four $Re_c = 100; 170; 200; 250$

Conclusion

Summary

- Study of the two-dimensional dynamics of the flow past a rotating cylinder.
- Identification of two *organizing centers*, a *TB-Cusp* bifurcation and a *Generalized Hopf* bifurcation.
- Existence of multiple steady-state solutions and a homoclinic connection near the TB-Cusp point.
- Existence of of a single limit cycle or two limit cycles in the vicinity of the GH point.

Codes for steady-state continuation on parameters, time-stepping simulations, linear stability, normal form reduction …are available in

https://gitlab.com/stabfem/StabFem

For more details, [5, 1]

Questions?

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