The O(2) equivariant Steady–Hopf interaction

Dynamics of the wake of axisymmetric objects (WFA) and in mixed convection (WFA-MC).

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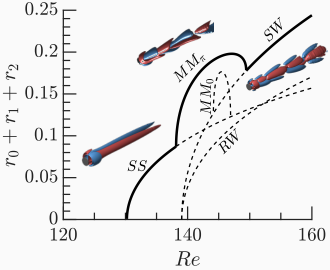
Motivation – Mode interaction steady-unsteady modes

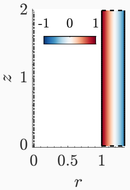


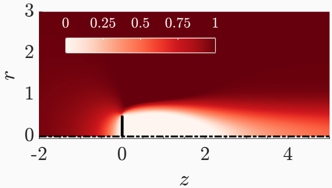
1. Mode interaction of a steady and two unsteady modes of a O(2) symmetric system.
    - Study of the successive bifurcations.
    - Existence of global bifurcations, e.g., robust heteroclinic cycles [1].
2. Applications:
    - Taylor Couette Flow (TCF)
    - Wake flow of axisymmetric objects (WFA) and in mixed convection (WFA-MC)
    - Axisymmetric rigid falling bodies (RFA) and rising bubble (RBA).

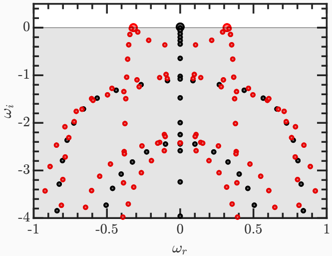
Motivation – Interaction between two linear stability modes (II)

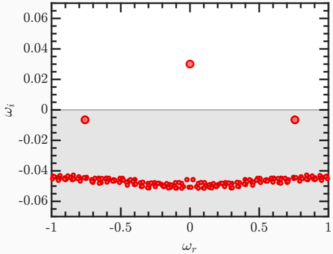
The flow state $\mathbf{q} = [\mathbf{u}, p, T]$, is decomposed as

$$\mathbf{q} = \mathbf{Q}_0 + \mathrm{Re}\left[a_0(t)\mathrm{e}^{-im_0\theta}\hat{\mathbf{u}}_s\right] + \mathrm{Re}\left[a_1(t)\mathrm{e}^{-im_1\theta}\hat{\mathbf{u}}_{h,-1} + a_2(t)\mathrm{e}^{im_1\theta}\hat{\mathbf{u}}_{h,1}\right]. \quad (1)$$









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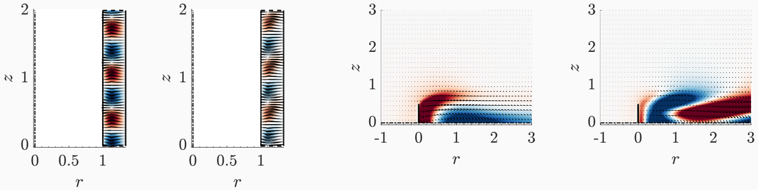
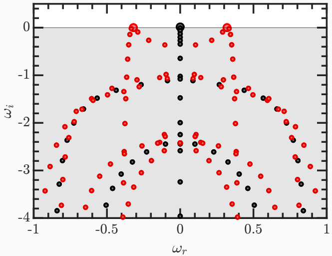
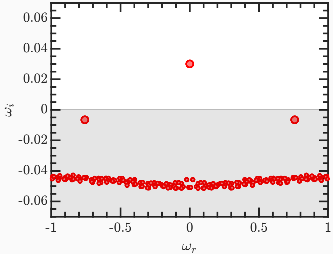






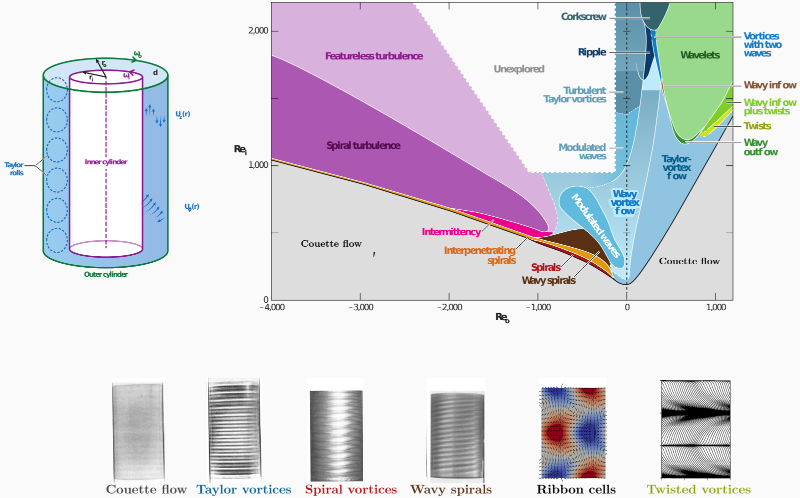
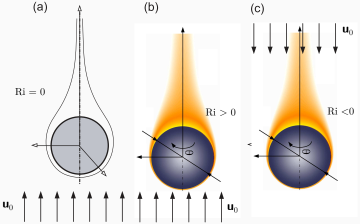


Figure 1: Phase portrait of the Taylor–Couette configuration. Courtesy of [4, 2, 5, 1]

Formulation

Governing equations

$$
\begin{aligned}
\mathbf{B}\frac{\partial \mathbf{q}}{\partial t} &= \mathbf{F}(\mathbf{q},\eta) \equiv \mathbf{L}\mathbf{q} + \mathbf{N}(\mathbf{q},\mathbf{q}) + \mathbf{G}(\mathbf{q},\eta), && \text{in } \Omega,\\
\mathbf{D}_{bc}\mathbf{q}(\mathbf{x}) &= \mathbf{q}_{\partial\Omega}, && \text{on } \partial\Omega\\
\text{with } \mathbf{L}\mathbf{q} &= \begin{pmatrix} -\nabla P \\ \nabla\cdot\mathbf{U} \\ 0 \end{pmatrix}, \quad \mathbf{N}(\mathbf{q}_1,\mathbf{q}_2) = -\begin{pmatrix} \mathbf{U}_1\cdot\nabla\mathbf{U}_2 \\ 0 \\ \mathbf{U}_1\cdot\nabla T \end{pmatrix} && \text{in } \Omega,\\
\mathbf{G}(\mathbf{q},\eta) &= \begin{pmatrix} \frac{1}{\mathrm{Re}}\nabla\cdot\left(\nabla\mathbf{U} + (\nabla\mathbf{U})^T\right) + \mathrm{Ri}\,T\mathbf{e}_z \\ 0 \\ \frac{1}{\mathrm{RePr}}\nabla^2 T \end{pmatrix} && \text{in } \Omega,
\end{aligned} \tag{3}
$$



Normal form reduction

The normal form reduction of the governing equations with the ansatz

$$
\begin{aligned}
\mathbf{q}(t,\tau) &= \mathbf{Q}_0 + \varepsilon \mathbf{q}_{(\varepsilon)}(t,\tau) + \varepsilon^2 \mathbf{q}_{(\varepsilon^2)}(t,\tau) + O(\varepsilon^3) \\
&\equiv \mathbf{Q}_0 + \mathrm{Re}\big(a_0(\tau) e^{-i m_0 \theta} \hat{\mathbf{q}}_0\big) \\
&+ \mathrm{Re}\big(a_1(\tau) e^{-i\omega t} e^{-i m_1 \theta} \hat{\mathbf{q}}_1 + a_2(\tau) e^{-i\omega t} e^{i m_2 \theta} \hat{\mathbf{q}}_2\big)
\end{aligned} \tag{4}
$$

where $\varepsilon \ll 1$ is a small parameter.

$$
\varepsilon \mathbf{B} \frac{\partial \mathbf{q}_{(\varepsilon)}}{\partial t} + \varepsilon^2 \mathbf{B} \frac{\partial \mathbf{q}_{(\varepsilon^2)}}{\partial t} + \varepsilon^3 \Big[\mathbf{B} \frac{\partial \mathbf{q}_{(\varepsilon^3)}}{\partial t} + \mathbf{B} \frac{\partial \mathbf{q}_{(\varepsilon)}}{\partial \tau} \Big] \tag{5}
$$

and the RHS respectively,

$$
\mathbf{F}(\mathbf{q}, \boldsymbol{\eta}) = \mathbf{F}_{(0)} + \varepsilon \mathbf{F}_{(\varepsilon)} + \varepsilon^2 \mathbf{F}_{(\varepsilon^2)} + \varepsilon^3 \mathbf{F}_{(\varepsilon^3)}. \tag{6}
$$

Normal form (complex form)

The normal form reduction of the governing equations with the ansatz

$$\begin{aligned}\mathbf{q}(t,\tau) &= \mathbf{Q}_0 + \varepsilon \mathbf{q}_{(\varepsilon)}(t,\tau) + \varepsilon^2 \mathbf{q}_{(\varepsilon^2)}(t,\tau) + O(\varepsilon^3) \\ &\equiv \mathbf{Q}_0 + \mathrm{Re}\big(a_0(\tau) e^{-im_0\theta} \hat{\mathbf{q}}_0\big) \\ &+ \mathrm{Re}\big(a_1(\tau) e^{-i\omega t} e^{-im_1\theta} \hat{\mathbf{q}}_1 + a_2(\tau) e^{-i\omega t} e^{im_2\theta} \hat{\mathbf{q}}_2\big)\end{aligned} \tag{7}$$

Dynamics are then reduced to a six dimensional subspace with the symmetries

$$\begin{aligned}\Phi &: (a_0, a_1, a_2) \to \big(a_0, a_1 e^{i\phi}, a_2 e^{i\phi}\big), \quad \kappa : (a_0, a_1, a_2) \to \big(\overline{a}_0, a_2, a_1\big) \\ R_\alpha &: (a_0, a_1, a_2) \to \big(a_0 e^{i\alpha}, a_1 e^{i\alpha}, a_2 e^{-i\alpha}\big)\end{aligned} \tag{8}$$

$$\begin{aligned}\dot{a}_0 &= \lambda_s a_0 + l_0 a_0 |a_0|^2 + l_1 \big(|a_1|^2 + |a_2|^2\big) a_0 \\ &+ i l_2 \big(|a_2|^2 - |a_1|^2\big) a_0 + l_3 \overline{a}_0 \overline{a}_2 a_1\end{aligned} \tag{9a}$$

$$\begin{aligned}\dot{a}_1 &= \big(\lambda_h + i\omega_h\big) a_1 + \big(B|a_1|^2 + \big(A+B\big)|a_2|^2\big) a_1 \\ &+ C a_1 |a_0|^2 + D a_0^2 a_2\end{aligned} \tag{9b}$$

$$\begin{aligned}\dot{a}_2 &= \big(\lambda_h + i\omega_h\big) a_2 + \big(B|a_2|^2 + \big(A+B\big)|a_1|^2\big) a_2 \\ &+ C a_2 |a_0|^2 + D \overline{a}_0^2 a_1\end{aligned} \tag{9c}$$

Normal form (polar form)

However, it is more convenient to work with the normal form in its polar form ($a_j = r_j e^{i\phi_j}$) for $j = 0, 1, 2$, and the phase $\Psi = \phi_1 - \phi_2 - 2\phi_0$

$$\dot{r}_0 = \Big[\lambda_s + l_0 r_0^2 + l_1 \big(r_1^2 + r_2^2\big)\Big] r_0 + l_3 r_0 r_1 r_2 \cos\Psi \tag{10a}$$

$$\dot{r}_1 = \Big[\lambda_h + B_r r_1^2 + (A_r + B_r) r_2^2 + C_r r_0^2\Big] r_1 + r_0^2 r_2 \big(D_r \cos\Psi + D_i \sin\Psi\big) \tag{10b}$$

$$\dot{r}_2 = \Big[\lambda_h + B_r r_2^2 + (A_r + B_r) r_1^2 + C_r r_0^2\Big] r_2 + r_0^2 r_1 \big(D_r \cos\Psi - D_i \sin\Psi\big) \tag{10c}$$

$$\dot{\Psi} = (A_i - 2 l_2)(r_2^2 - r_1^2) - 2 l_3 r_1 r_2 \sin\Psi + r_0^2 D_i \cos\Psi \left[\frac{r_2}{r_1} - \frac{r_1}{r_2}\right] - r_0^2 D_r \sin\Psi \left[\frac{r_2}{r_1} + \frac{r_1}{r_2}\right] \tag{10d}$$

which allows us to reduce dynamics to a four dimensional subspace (slicing the two continuous symmetries, now discrete).

Normal form reduction (Order 0 & 1)

The zeroth order $\mathbf{Q}_0$ of the reduction procedure is the steady state equation evaluated at the threshold of instability, i.e. $\boldsymbol{\eta} = \mathbf{0}$,

$$\begin{aligned} \mathbf{0} &= \mathbf{F}(\mathbf{Q}_0, \mathbf{0}),\ \mathbf{x} \text{ in } \Omega, \\ \mathbf{D}_{bc}\mathbf{Q}_0(\mathbf{x}) &= \mathbf{Q}_{0,\partial\Omega},\ \mathbf{x} \text{ on } \partial\Omega. \end{aligned} \tag{11}$$

The first order solution $\mathbf{q}_{(\varepsilon)}(t,\tau)$ is composed of the eigenmodes of the linearized system

$$\mathbf{q}_{(\varepsilon)}(t,\tau) \equiv \mathrm{Re}\big(a_0(\tau)e^{-im_0\theta}\hat{\mathbf{q}}_0\big) + \mathrm{Re}\big(a_1(\tau)e^{-i\omega t}e^{-im_1\theta}\hat{\mathbf{q}}_1 + a_2(\tau)e^{-i\omega t}e^{im_2\theta}\hat{\mathbf{q}}_2\big) \tag{12}$$

where the reflection symmetry of $O(2)$ imposes $m_2 = -m_1$. Each term $\hat{\mathbf{q}}_\ell$ of the first order expansion is a solution of the following linear equation

$$\begin{aligned} \mathbf{J}_{(\omega_\ell, m_\ell)}\hat{\mathbf{q}}_\ell &= \left(i\omega_\ell\mathbf{B} - \frac{\partial\mathbf{F}}{\partial\mathbf{q}}\Big|_{\mathbf{q}=\mathbf{Q}_0,\boldsymbol{\eta}=\mathbf{0}}\right)\hat{\mathbf{q}}_\ell,\ \mathbf{x} \text{ in } \Omega, \\ \mathbf{D}_{bc}\hat{\mathbf{q}}_\ell(\mathbf{x}) &= 0,\ \mathbf{x} \text{ on } \partial\Omega. \end{aligned} \tag{13}$$

where $\frac{\partial\mathbf{F}}{\partial\mathbf{q}}\big|_{\mathbf{q}=\mathbf{Q}_0,\boldsymbol{\eta}=\mathbf{0}}\hat{\mathbf{q}}_\ell = \mathbf{L}_{m_\ell}\hat{\mathbf{q}}_\ell + \mathbf{N}_{m_\ell}(\mathbf{Q}_0, \hat{\mathbf{q}}_\ell) + \mathbf{N}_{m_\ell}(\hat{\mathbf{q}}_\ell, \mathbf{Q}_0)$.

Normal form reduction (Order 2)

Non-linear second order terms in ε are

$$\begin{aligned}\mathbf{F}_{(\varepsilon^2)} \equiv & \sum_{j,k=0}^{2}\left(a_j a_k \mathbf{N}(\hat{\mathbf{q}}_j,\hat{\mathbf{q}}_k) e^{-i(m_j+m_k)\theta} e^{-i(\omega_j+\omega_k)t} + \text{ c.c.}\right) \\ & + \sum_{j,k=0}^{2}\left(a_j \overline{a}_k \mathbf{N}(\hat{\mathbf{q}}_j,\overline{\hat{\mathbf{q}}}_k) e^{-i(m_j-m_k)\theta} e^{-i(\omega_j-\omega_k)t} + \text{ c.c.}\right) \\ & + \sum_{\ell=0}^{2}\eta_\ell \mathbf{G}(\mathbf{Q}_0,\mathbf{e}_\ell),\end{aligned} \tag{14}$$

The second order term can be expanded as follows

$$\mathbf{q}_{(\varepsilon^2)} \equiv \sum_{\substack{j,k=0\\ k\leq j}}^{2}\left(a_j a_k \hat{\mathbf{q}}_{j,k} + a_j \overline{a}_k \hat{\mathbf{q}}_{j,-k} + \text{ c.c }\right) + \sum_{\ell=0}^{2}\eta_\ell \mathbf{Q}_0^{(\eta_\ell)}, \tag{15}$$

Finally, second-order terms are computed by solving

$$\begin{aligned}\mathbf{J}_{(\omega_j+\omega_k, m_j+m_k)}\hat{\mathbf{q}}_{j,k} &= \hat{\mathbf{F}}^{(j,k)}_{(\epsilon^2)}, \\ \mathbf{J}_{(0,0)}\mathbf{Q}_0^{(\eta_\ell)} &= \mathbf{G}(\mathbf{Q}_0,\mathbf{e}_\ell).\end{aligned} \tag{16}$$

Normal form reduction (Order 3)

The linear terms λ_s and λ_h are determined as follows

$$\lambda_s = \frac{\langle \hat{\mathbf{q}}_0^\dagger, \hat{\mathbf{F}}^{(a_0)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_0^\dagger, \mathbf{B}\hat{\mathbf{q}}_0 \rangle}, \ \lambda_h = \frac{\langle \hat{\mathbf{q}}_1^\dagger, \hat{\mathbf{F}}^{(a_1)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_1^\dagger, \mathbf{B}\hat{\mathbf{q}}_1 \rangle} = \frac{\langle \hat{\mathbf{q}}_2^\dagger, \hat{\mathbf{F}}^{(a_2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_2^\dagger, \mathbf{B}\hat{\mathbf{q}}_2 \rangle}. \tag{17}$$

The real cubic coefficients l_i for $i = 0, 1, 2, 3$ are obtained as

$$\begin{aligned} l_0 &= \frac{\langle \hat{\mathbf{q}}_0^\dagger, \hat{\mathbf{F}}^{(a_0|a_0|^2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_0^\dagger, \mathbf{B}\hat{\mathbf{q}}_0 \rangle}, & l_3 &= \frac{\langle \hat{\mathbf{q}}_0^\dagger, \hat{\mathbf{F}}^{(\overline{a}_0 a_1 \overline{a}_2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_0^\dagger, \mathbf{B}\hat{\mathbf{q}}_0 \rangle} \\ l_1 - il_2 &= \frac{\langle \hat{\mathbf{q}}_0^\dagger, \hat{\mathbf{F}}^{(a_0|a_1|^2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_0^\dagger, \mathbf{B}\hat{\mathbf{q}}_0 \rangle}, & l_1 + il_2 &= \frac{\langle \hat{\mathbf{q}}_0^\dagger, \hat{\mathbf{F}}^{(a_0|a_2|^2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_0^\dagger, \mathbf{B}\hat{\mathbf{q}}_0 \rangle}. \end{aligned} \tag{18}$$

Finally, the complex coefficients A, B, C and D are computed as follows,

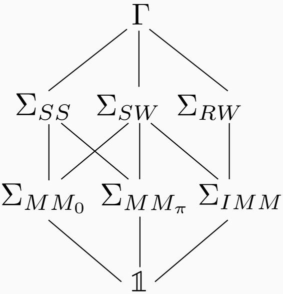
$$\begin{aligned} B &= \frac{\langle \mathbf{q}_1^\dagger, \hat{\mathbf{F}}^{(a_1|a_1|^2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_1^\dagger, \mathbf{B}\hat{\mathbf{q}}_1 \rangle}, & A + B &= \frac{\langle \mathbf{q}_1^\dagger, \hat{\mathbf{F}}^{(a_1|a_2|^2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_1^\dagger, \mathbf{B}\hat{\mathbf{q}}_1 \rangle}, \\ C &= \frac{\langle \mathbf{q}_1^\dagger, \hat{\mathbf{F}}^{(a_1|a_0|^2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_1^\dagger, \mathbf{B}\hat{\mathbf{q}}_1 \rangle}, & D &= \frac{\langle \mathbf{q}_1^\dagger, \hat{\mathbf{F}}^{(a_0^2 a_2)}_{(\epsilon^3)} \rangle}{\langle \hat{\mathbf{q}}_1^\dagger, \mathbf{B}\hat{\mathbf{q}}_1 \rangle}. \end{aligned} \tag{19}$$

Unfolding of the normal form

Isotropy lattice – Primary bifurcations

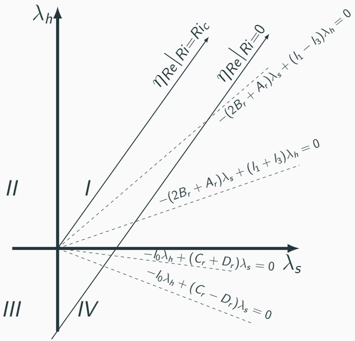
Name	Representative	Iso. group (complex)	Iso. group (polar)	Frequencies
Pure modes:				
TS	$(0,0,0,nd)$	$O(2)\times S^1$	$D_4 \rtimes \mathbb{Z}_2(\kappa)$	0
SS	$(r_a,0,0,nd)$	$\mathbb{Z}_2(\kappa)\times S^1$	$\mathbb{Z}_2(\kappa)\times\mathbb{Z}_2(\Phi_\pi)$	0
RW	$(0,r_a,0,nd)$	$\widetilde{SO(2)}$	$\mathbb{Z}_4(R_{\pi/2}\Phi_{\pi/2})$	1
SW	$(0,r_a,r_a,nd)$	$\mathbb{Z}_2(\kappa)\times\mathbb{Z}_2(R_\pi\Phi_\pi)$	$\mathbb{Z}_2(\kappa)\times\mathbb{Z}_2(R_\pi\Phi_\pi)$	1
Mixed modes:				
MM_0	$(r_a,r_b,r_b,0)$	$\mathbb{Z}_2(\kappa)$	$\mathbb{Z}_2(\kappa)$	1
MM_π	(r_a,r_b,r_b,π)	$\mathbb{Z}_2\left(\kappa\cdot R_\pi\Phi_\pi\right)$	$\mathbb{Z}_2\left(\kappa\cdot R_\pi\Phi_\pi\right)$	1
IMM	$(0,r_a,r_b,\Psi)$	$\mathbb{Z}_2(R_\pi\Phi_\pi)$	$\mathbb{Z}_2(R_\pi\Phi_\pi)$	1
Precessing waves:				
General	(r_a,r_b,r_c,Ψ)	$\mathbb{1}$	$\mathbb{1}$	2
Type A	(r_a,r_b,r_b,Ψ)	$\mathbb{1}$	$\mathbb{1}$	2
Type B	$(r_a,r_b,r_c,0 \text{ or } \pi)$	$\mathbb{1}$	$\mathbb{1}$	2
Type C	$(r_a,r_b,0,\Psi)$	$\mathbb{1}$	$\mathbb{1}$	2

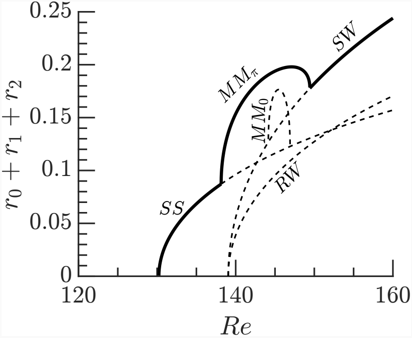
Nomenclature in different problems



Name	Name (TC)	Name (WFA)
TS	Taylor Couette Flow	Axisymmetric state
SS	Taylor Vortex Flow	Steady shedding
SW	Ribbon cells	Standing wave
RW	Spiral vortex	Spiral Shedding state
MM_0	Twisted vortices	Reflection Symmetry Preserving
MM_π	Wavy vortices	Reflection Symmetry Breaking
IMM	Wavy Spirals	(-)
PrW	(-)	(-)

Construction of the bifurcation diagram



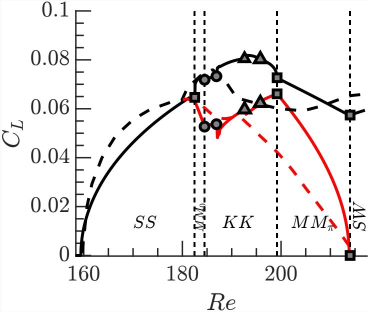


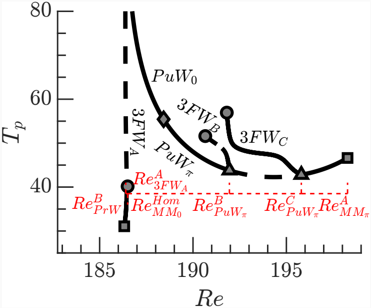
Modulated bifurcations

Name	Representative	Isotropy group	Frequencies
$\widetilde{MM}_{0,\pi}$	$(r_a(t), r_b(t), r_b(t), 0 \text{ or } \pi)$	$\mathbb{1}$	2
$\widetilde{IMM}$	$(0, r_b, r_c, \Psi(t))$	$\mathbb{1}$	2
PuW	$(r_a(t), r_b(t), r_c(t), \Psi(t))$ with $\overline{r}_b = \overline{r}_c$ and $\overline{\sin\Psi} = 0$	$\mathbb{1}$	2
3-frequency waves: (3FW)			
General	$(r_a(t), r_b(t), r_c(t), \Psi(t))$	$\mathbb{1}$	3
Type A	$(r_a(t), r_b(t), r_b(t), \Psi(t))$ with $\overline{\sin\Psi} \neq 0$	$\mathbb{1}$	3
Type B	$(r_a(t), r_b(t), r_c(t), 0 \text{ or } \pi)$ with $\overline{r}_b \neq \overline{r}_c$	$\mathbb{1}$	3
Type C	$(0, r_b(t), r_c(t), \text{nd})$ with $\overline{r}_b \neq \overline{r}_c$	$\mathbb{1}$	3
Type D	$(r_a(t), r_b(t), 0, \Psi(t))$ with $\overline{\sin\Psi} \neq 0$	$\mathbb{1}$	3

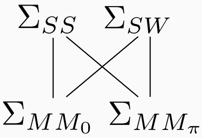
The quasiperiodic state $\widetilde{MM}_{0,\pi}$ is known as Modulated Wavy Vortex Flow (MWVF) in the Taylor Couette problem.

A more complex bifurcation diagram





Robust (asymptotically stable) heteroclinic cycles



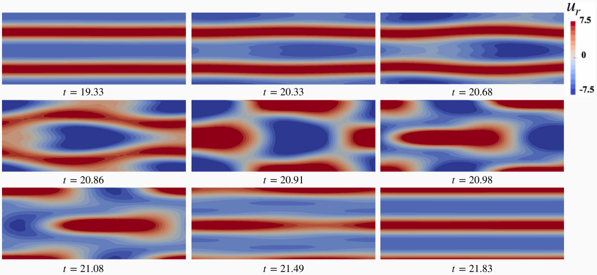
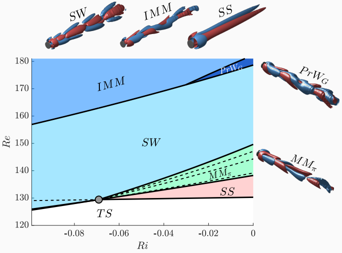
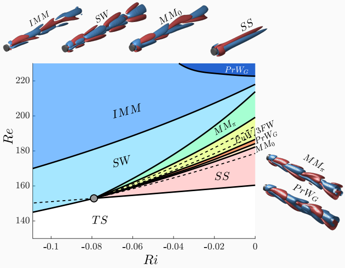


Figure 2: Courtesy of [1]. Identified in the region of Wavy Spirals (IMM), near the boundary of Modulated Wavy Vortex ($\widetilde{MM}_{0,\pi}$). The physical mechanism is SSP [3]

Summary

Summary





- Study of the bifurcation scenario for the steady-Hopf mode interaction with O(2) symmetry.
- Applications to a large variety of flows, Taylor–Couette, wake axisymmetric flows, falling rigid bodies, etc.

Codes for steady-state continuation on parameters, time-stepping simulations, linear stability, normal form reduction . . . are (or will be) available in

`https://`
`//gitlab.com/stabfem/StabFem`

Questions?

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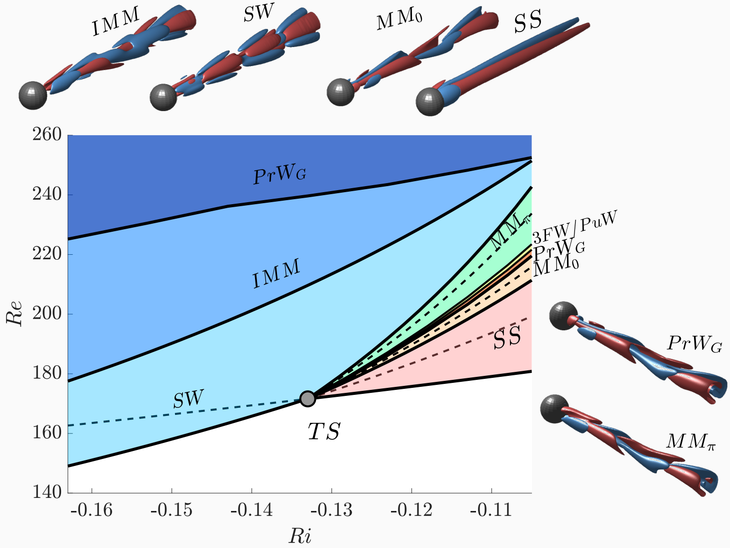
Sphere



IMM
SW
MM_0
SS
PrW_G
IMM
SW
TS
MM_π
3FW/PuW
PrW_G
MM_0
SS
PrW_G
MM_π
260
240
220
200
180
160
140
Re
-0.16
-0.15
-0.14
-0.13
-0.12
-0.11
Ri